Abstract

Dynamic modelling of components and systems is a key stage in the process of design and validation of any machine capable of supporting time variable load conditions. Hybrid modelling techniques, which combine information obtained from the experimental field with the coming of numerical discretization methods, are the most used to complete reliable and representative models of components and assemblies analyzed. This article describes the process of characterization in systems of gears as part of more elaborate power trains. The characterization of the complete system is carried out from individual dynamic models of each component and using modal synthesis techniques the entire model is obtained. Modal synthesis is defined assembling into the system of equations that solves the full dynamic problem, the modal vectors of each subset in combination with the modeling of joints between them and applying properties of orthogonality of modal vectors up to get the appropriate approach for the resolution of the numerical system. Obtaining as a result the natural frequencies and modal vectors of the full system. As a complement to the characterization of the whole process, and framed within the scope of the stages of design and validation of the systems on which applies the technique of modal synthesis, the article describes methods of correlation between experimental and theoretical results and applications of modal model to the calculation of different operating conditions.

The Pair of Gears as a Source of Vibrations. Transmission Error and Subsequent Propagation of Vibrations.

Each pair of gears can be considered as a source of vibrations. This excitation is the result of a force that varies as well in its magnitude or its point of application or direction. For example, type Wildhaver-Novikov gears have associated high levels of vibration due to the variation of the point of application of the resultant, which moves axially as the contact area is moved along the line of mesh. In comparison, profile of involute gears, do not modify its direction except for friction effects but the amplitude of the resulting interaction, due either to deviations from the theoretical involute profile or to the effect of the variable elasticity of the tooth at different stages of the mesh.

„Transmission Error“ (TE) is used in order to define and quantify the manufacturing and design “errors” that causes the gear pair not to be capable of transmitting continuously torque. Assuming that the input shaft is driven with constant angular speed, it is
Fig. 1: Transmission Error

Fig. 2: Vibration path

Fig. 3: SFT results
expected that the output shaft keep the condition of stable rotational speed regime. Any variation in such rate causes a discrepancy regarding the “correct” position of the output shaft, understood as a transmission error, resulting in the generation of vibrations of gear. Thus, formally the transmission error is defined as the difference between the position that would occupy the output shaft if the transmission were perfect and the actually occupied by result of their design, manufacturing and Assembly process parameters.

The vibration generated by the pair of engagement and quantified using the transmission error is transmitted through the rest of structure to structure-borne noise. Normally roller bearings as elements responsible for sustaining the pair of gear, are the first items to receive excitation in the form of vibration, and they react transmitting it through the roller elements to the fixed races and from there to the rest of the structure. At this point is where the distribution of mass, stiffness and damping of the powertrain and its casing play an important role in defining the dynamics of the structure and therefore the mechanism of amplification or attenuation that vibrations generated in the gear suffer. Given the issues raised, there are different paths to optimize the various mechanical components from the point of view of noise and vibration:

- Gearmesh optimization as a source of vibration
- Reduction of dynamic transmission of vibrations from the contact to the radiant panels
- Noise and vibration absorption

**Vibrations of Gear. Parameters Design and Manufacturing for its Optimization**

Some of the main parameters of design and manufacturing that have to be considered for the optimization of gear pairs are listed below. In general, to reduce the gear vibration, the precision of gears must be assured by modifications of the involute profile, tight manufacturing tolerances and improvements in the surface of the teeth. The design parameters include:

- **Contact ratio** is the key parameter. As contact ratio increases, the load transmitted by each tooth individually is reduced, thus reducing the dynamic forces generated at the entrance and exit of each tooth in the couple’s engagement. Besides, as it increases the number of teeth in contact, manufacturing individual errors are averaged with the rest and therefore decrease.

- **Tooth shape:** A slender tooth is associated with greater elasticity, but also results in a higher ratio of contact and less intensity of impact at the entrance of the tooth. Similarly, the face width also involves improvements in the contact ratio but makes the manufacturing process more difficult.

- **Profile changes:** In order to avoid interference that may result from the deflections on teeth, shafts and housings, in certain load cases, changes to the profile from the theoretical involute, whether in the tip or at the tooth root, may be implemented although compromise solutions must be reached that prevent impacts by interference without diminishing significantly the contact ratio.

DEWI carries out measurements and evaluations to determine the electrical characteristics of single wind turbines, wind farms, solar inverters and of other generation units/systems according to the currently applicable standards (e.g. IEC 61400-21) and is accredited by “German Accreditation Body” (DAkkS) in line with EN ISO/IEC 17025:2005 and member of MEASNET.

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Backlash adjustment: a clearance must be defined between teeth enough under all conditions of operation to avoid gear with interference but without causing excessive vibration levels by impact. Other parameters such as angle of pressure, the module or the own generation tool play an important role, usually indirectly through its influence on the contact ratio, to define the correct compromise between reliability and softness of the gearmesh. In order to quantify the transmission error, (TE), the most appropriate test is the Single Flank Test (SFT). It is completed by driving a gearmesh at low speed of rotation and transmitted torque, while measuring with two high resolution encoders the input and output shaft position in order synthesize the TE:

\[
TE = R_{b1} \left[ \theta_1 - \left( \frac{N_2}{N_1} \right) \theta_2 \right]
\]

being \( \theta_1 \) y \( \theta_2 \) the angular position of the input and output shaft, \( N_1 \) y \( N_2 \) the respective number of teeth and \( R_{b1} \) the radius of the base circumference for the first gear.

The following parameters are obtained as a result of the test:
- Parameters related to contact error: tooth to tooth
  - Gearmesh harmonics: normally 25 \( \mu \)rad maximum for the first gear mesh harmonic
  - Average profile Error: normally \( \pm 20 \mu \)rad (40 \( \mu \)rad average total peak-to-peak)
- Parameters relating to mounting between pinion and Crown related errors:
  - Harmonics of rotation: normally 150 \( \mu \)rad maximum for the first rotation of the crown being lower the values for the harmonic of the pinion rotation and the subsequent harmonic.

As a complement to the SFT and in order to obtain experimental data that can be contrasted with the virtual model to be described in the following sections, it is advisable to carry out a series of measurements of vibrations or sound on key points during operation conditions. Spectral analysis and order tracking in frequency or angular domain, Campbell diagrams and other representations play a key role in identifying the effects really associated with gear pairs in rotating machines.

**Vibration Transmission. A Methodology for Discretization of Complex Mechanical Systems.**

Once characterized the vibration source, it is required to evaluate the system or component responsible for its transmission from the dynamic point of view. The modal synthesis technique allows you to tackle the problem of splitting it between different subsystems, so characterizing the dynamics of each of them on free boundary conditions and the boundary condition between each pair of sets in a separate way the full dynamic model of the system can be achieved. This procedure shows significant advantages over direct characterization:

- The dynamic model of each subsystem can be obtained through an experimental approach, EMA, or through a theoretical approach, FEM. Being able to combine later subsystems of each type.
Each subsystem can be represented by a more accurate and refined models.
Any structural modification that should be undertaken at any stage of development will only affect the model of the subsystem relevant, so the full system can be updated faster.

There is the possibility of generating ‘libraries of standard components’ for which have been created models of proven reliability.

As stated, to formulate the problem of coupling using modal synthesis, two subsystems which are characterized by their normal modes on free boundary condition are considered to be elastically coupled. Based on the property of orthogonality between modes, the following relationship can be established for each subsystem.

\[
\begin{bmatrix}
\{u_i\} \\
\{u_e\}
\end{bmatrix}
= \begin{bmatrix}
\phi_{im} \\
\phi_{en}
\end{bmatrix} \cdot \{p_{im}\} = [\phi_m] \cdot \{p_m\}
\]

Where
- \([\phi_m]\) Matrix containing the normal modes (including rigid body modes) of the free system
- \([p_m]\) Factor of participation of each normal mode in the composition of the deformed.

The elastic connection between both sub-systems fullfills:
- Action - Reaction principle: \(\{f_i\} = -\{f_e\}\)
- Flexible parameters:

\[
\begin{bmatrix}
\{u_i\} \\
\{u_e\}
\end{bmatrix}
= [K_{op}] \cdot \begin{bmatrix}
\{u_i\} \\
\{u_e\}
\end{bmatrix} \quad \text{where} \quad [K_{op}] = \begin{bmatrix}
[K_{ii}] & -[K_{ie}] \\
-[K_{ei}] & [K_{ee}]
\end{bmatrix}
\]

In this way by attaching both sub-systems, general Dynamics equation can be formulated as:

\[
\begin{bmatrix}
[M_i] & [M_e] \\
[M_e] & [M_e]
\end{bmatrix}
\begin{bmatrix}
\{\ddot{u}_i\} \\
\{\ddot{u}_e\}
\end{bmatrix}
+ \begin{bmatrix}
[K_i] & [K_e] \\
[K_e] & [K_e]
\end{bmatrix}
\begin{bmatrix}
\{u_i\} \\
\{u_e\}
\end{bmatrix}
= \begin{bmatrix}
\{f_i\} \\
\{f_e\}
\end{bmatrix}
\]

Making the switch to modal coordinates and introducing the characteristics of the elastic union the following eigenvalue problem giving as a result the eigenvalues and eigenvectors of the complete system:

\[
\begin{bmatrix}
[M_{op}] & [M_{op}] \\
[M_{op}] & [M_{op}]
\end{bmatrix}
\begin{bmatrix}
\{\ddot{\phi}_m\} \\
\{\ddot{\phi}_m\}
\end{bmatrix}
+ \begin{bmatrix}
[K_{op}] & [K_{op}] \\
[K_{op}] & [K_{op}]
\end{bmatrix}
\begin{bmatrix}
\{\phi_m\} \\
\{\phi_m\}
\end{bmatrix}
= \begin{bmatrix}
\{0\} \\
\{0\}
\end{bmatrix}
\]

Fig. 6: Virtual vs Experimental forced response comparison
Modal Model Applications

Once dynamic model is obtained and validated, modal shapes and eigenvalues are used as the basis for different processing and analysis. As an example, from the data obtained from the single flank test, an analysis of the forced response that allows to know vibration levels in casing check points and compare them with those measured experimentally in operating conditions equivalent to those used for the single flank test. The following expression contains in the frequency domain response in terms of displacement for the whole structure characterized by \( r \) natural modes and subjected to a deterministic excitation.

\[
\{x(t)\} = \{r\} e^{i\omega t} = \sum_{r=1}^{r} \left( \begin{array}{c} \varphi_r \end{array} \right) \frac{1}{\sqrt{2\pi}} \left( \begin{array}{c} F_r \end{array} \right) e^{i\omega t}
\]

Conclusion

Based on the described methodology and the results obtained it is concluded:

- The parameters of quality and design in the manufacturing process and assembly gear trains have a significant effect on the level of noise and vibration generated by components that use gears for the transmission of torque. The Single Flank test as well as operational measures are useful tools when it comes to parameterize the quality of the gear mesh, being necessary to consider, not only its durability, but also its dynamic behaviour from early stages of design.
- The vibrations caused by the pair of gears is transmitted through the structures inherent to each component, and therefore such structures can be “tuned” as if it were at the time filters considering generated vibration propagation. It is necessary to have a dynamic system model to optimize the design.
- The modal analysis is the best choice for the dynamic model of mechanical systems. The associated natural frequencies and modal forms are obtained as a result of giving relevant information jointly from the point of view of behaviour of the structure and analysis providing the mathematical basis for subsequent analysis to varying loads or stages of design optimization.
- Modal synthesis techniques simplify the process of obtaining the dynamic model at the time provided the basis for the combination of theoretical and experimental models.

References

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